EE231E Project: Finite Traceback Viterbi Decoder

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Introduction

This report outlines the simulation of a finite traceback Viterbi decoder for the rate ½, 64-state convolutional encoder given by the following generator matrix:

$$G(D) = [D^6+D^5+D^3+D^2+1 \ , \ D^6+D^3+D^2+D+1].$$

The following sections describe the operation and performance of software written to characterize the code in terms of its d_{free}, L_d, N, and N_b , and to simulate its BER performance as a function of received SNR.

Code Discussion

The code for computing d_{free}, L_d, N, and N_b was written in C and compiled under gcc, as was the code for the encoder, decoder, and AWGN channel model. Octave, a code-compatable MATLAB alternative, was used to generate all plots as well as to calculate the transfer function bound. This section will discuss, one by one, the operation of all major routines used in the simulation.

main (C code)

The main routine is responsible for continuously generating blocks of data and applying the transmitter, channel, and receiver simulations until a specified number of bit errors has been met or exceeded. This process is repeated for each SNR of interest. High SNRs can cause the simulation to run for multiple hours, so intermediate results are printed after each simulated block so that the user can monitor the simulation's progress.

convstats (C code)

The convstats routine begins by generating 2D global arrays describing valid state transitions and the associated output weight. For example, the array of valid transitions has a 1 in position ij if the transition to state i from state j is possible for the given encoder. Once these arrays are generated, d_{free}, L_d, N, and N_b are generated by traversing the trellis from the zero state until all path metrics are greater than that of the zero state. This stopping algorithm only works for non-catastrophic codes, and so the simulation software should only be used for codes known to be non-catastrophic (such as the one provided for this project).

In order to test the accuracy of this routine, it was run for the $G(D) = [1+D+D^3, 1+D+D^2+D^3]$ example from Lecture 7. d_{free}, L_d, N, and N_b were all correctly identified (6, 11, 1, and 2, respectively).

encode (C code)

Prior to encoding the message, the encoder adds a number of zeros equal to the traceback depth to the beginning and end of the data message. This helps to

ensure that the trellis starts and ends in the zero state. Although this adds some processing overhead to the system, in general the traceback depth is negligible in comparison to the message length, and so it doesn’t have a noticeable effect on the simulation runtime.

addAWGN (C code)

The addAWGN function simulates the noisy channel by adding AWGN of a specified power to the input signal (the transmitter output). Noise samples are generated via the central limit theorem: a large number of randomly generated binary numbers are averaged to produce a zero-mean Gaussian random variable.

This seemingly straightforward function actually caused significant difficulty during BER simulation. The simulation originally averaged 128 points to create the Gaussian noise, but the resultant simulated BERs were consistently slightly above the transfer function bound for input SNRs greater than approximately 4dB. After significant troubleshooting, it was discovered that increasing the number of samples used to create the AWGN to 256 virtually eliminated this excess BER. This shows that the simulation is very sensitive to the quality of AWGN. Troubleshooting this issue was the most difficult part of the project, but it was also interesting to discover that the noise quality could hurt BER so noticeably.

Because 256 points are used to generate noise for each transmitted symbol, a great deal of computation is needed just to generate noise. A future version of this simulation software would benefit from a faster, optimized AWGN generating function.

decode (C code)

The decoder keeps track of the minimum path to each state in a $64 \times d_{free}$ circular buffer. When decoding a bit, the buffer is traversed until arriving at the column of the oldest stored state, at which point the LSB of corresponding state (i.e. the value of the transition that caused the trellis to transfer to that state) is used as the output of the decoder. Using a circular buffer means that updating the buffer for each new input only requires overwriting a single column instead of re-arranging the entire data structure. After decoding the entire received signal, the first d_{free} bits are discarded because they were added as padding (see *encoder* description).

Because all previous routines had already been verified (except for the AWGN, which, as previously mentioned, was slightly suboptimal), the decoder was tested together with all other functions. The system was tested by generating a BER curve for the [7,5] octal code discussed by Krishna Sankar at http://www.dsplog.com/2009/01/14/soft-viterbi/. The simulated BER curve (Fig. 1) matched Mr. Sankar's (Fig. 2), and so the simulation software was deemed a success.

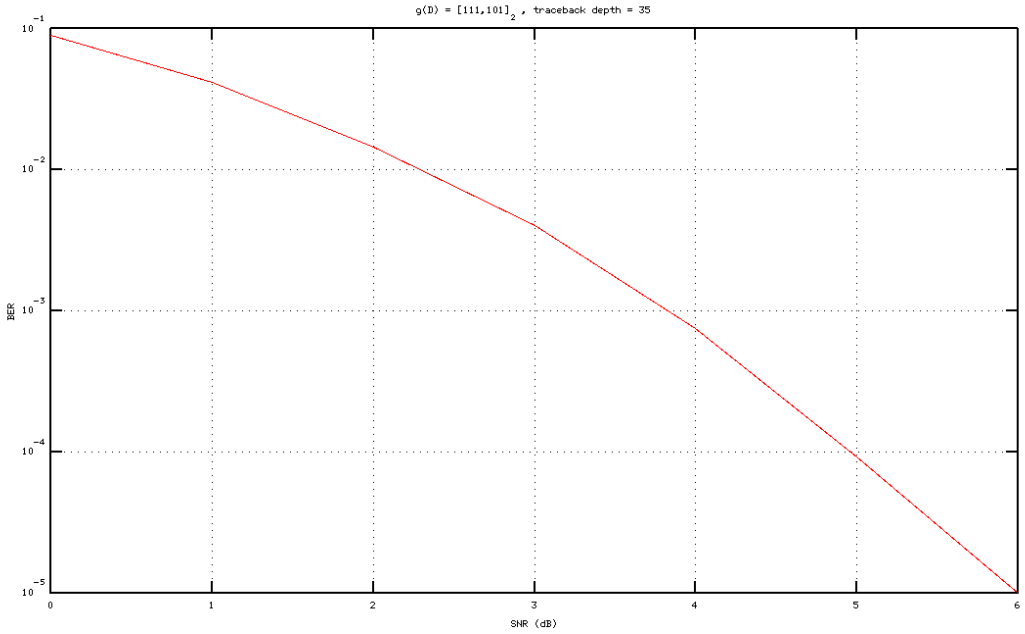

Figure 1: [7,5] octal code simulation results

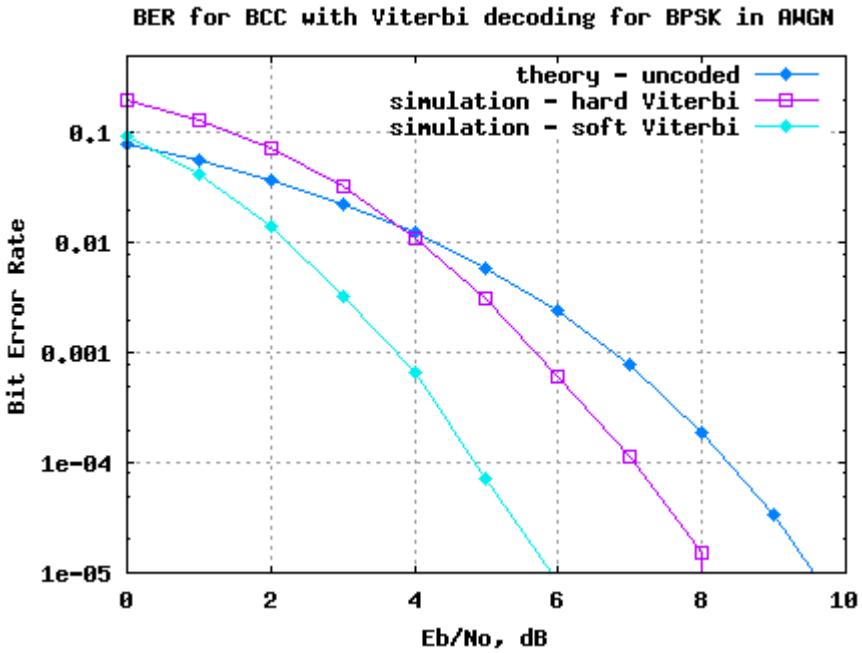

Figure 2: [7,5] octal code reference results (soft Viterbi curve), by Krishna Sankar

calc_transfer_bound (MATLAB function)

The code used to generate the transfer function bound was written as a MATLAB script so as to take advantage of the ease with which MATLAB/Octave can manipulate matrices. This function makes use of the *w* and *i* exponents of the T(W,I) matrix, generated and exported as matrices by the *convstats* C function.

Results

The simulation indicates that the code of interest has the following parameters:

- $d_{free} = 10$
- $L_d = 28$
- $N = 11$
- $N_b = 36$

The plot of BER vs. SNR is shown on the following page (Fig. 3) for a variety of traceback depths. All points are simulated in transmission blocks of 100000 bits until at least 100 errors are generated. The AWGN is generated using 256 samples for all SNRs below 4dB, and 512 samples for all SNRs above 4dB. The increase in points is due to the observation that the curves became more sensitive to the quality of the AWGN as BER decreased.

As seen in the plot, increasing the traceback depth makes significant performance improvements until approximately a depth of 35. There are still improvements as the traceback depth increases to twice d_{free} (56) and beyond, but the improvements become increasingly small.

As expected, the BER approximation and transfer function bound converge as SNR increases. The simulation results in the 4-5dB range are centered around the transfer function bound with some values above it and some below. The variation about the curve should go to zero as the number of errors simulated at each SNR increases, but as the SNR approaches 5dB and beyond, the simulations start taking a long time. It becomes impossible to simulate 100 errors in a timely manner after 5 dB, and any higher SNRs require multiple hours of computation. Luckily, the approximation curve and transfer function bound are shown to become more accurate as SNR increases, so simulation can be avoided in favor of using these easily computed theoretical values.

Conclusion

The simulation was a success. The simulated results show agreement with both the BER approximation and the transfer function bound at increasing SNR. The software described in this report would be a useful tool for further simulation and investigation of non-catastrophic finite traceback Viterbi decoders.

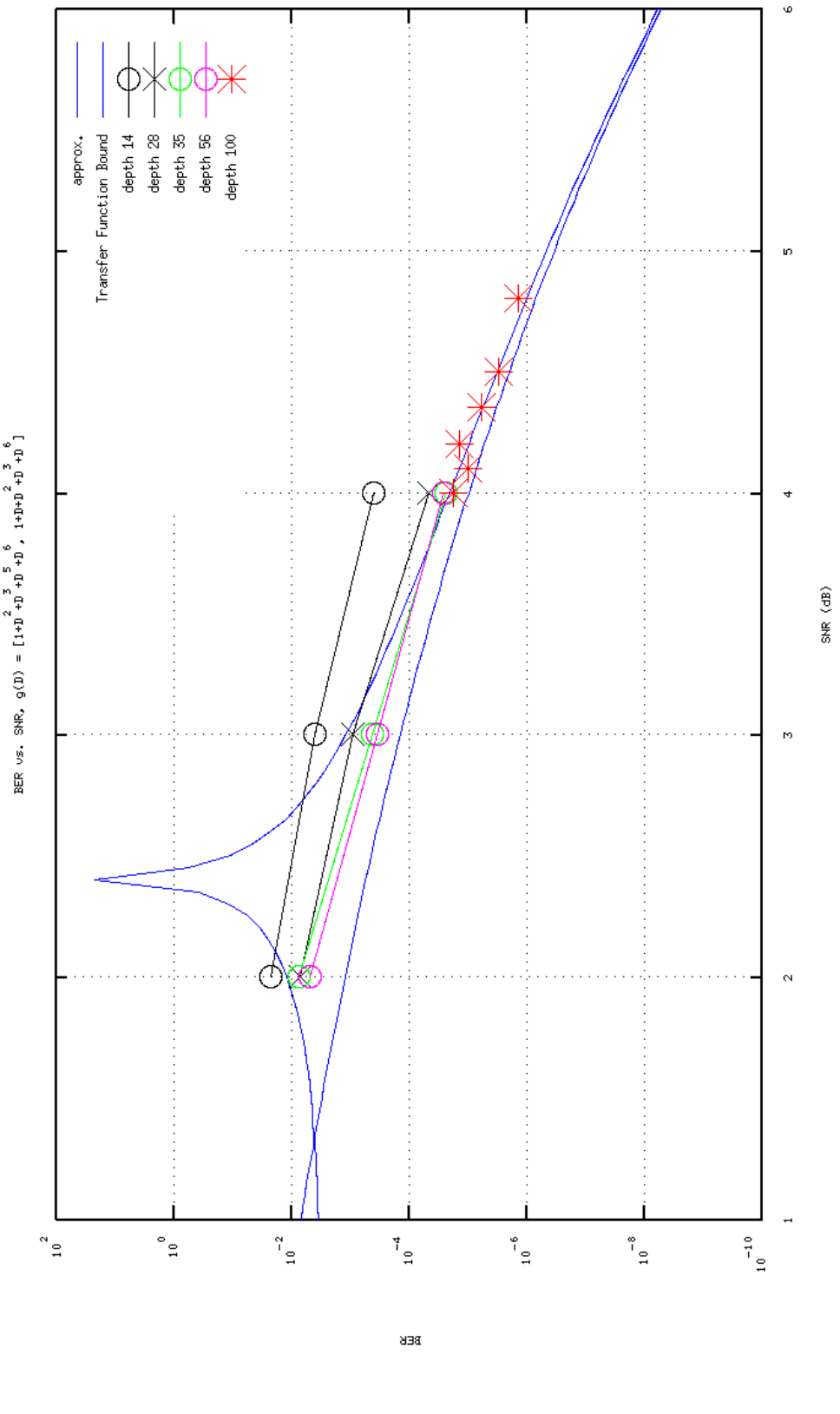

Figure 3: BER vs. SNR plot for code of interest

Code Listing

1) Main Code (everything except transfer bound calculation, plotting routines)

```
//ee231e.c
//Generate dfree, Ld, N, and Nb for a specified conv. code
//of order 1/n
//EE231E project
//Alex McAuley
//Apr. 28, 2012
//amcauley@ucla.edu
//compilation: gcc -lm -o ee231e ee231e.c

#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <time.h>

/*SIMULATION PARAMETERS*/
#define Nin 100000
#define NtargetERR 100
#define tracebackdepth 100
#define NTESTSNRS 1
#define TESTSNRS {4.35}
#define order 6//2
#define nstates 64//4
#define orderplus1 (order+1)
#define orderminus1 (order-1)
#define npolys 2
#define Nrandn 512
#define FASTMSG 0 //don't generate new message each time, only new noise
#define GEN_TWI 0 //print out I and W matrices for external T(W,I) processing

#define DISPLAYSTATS 0 //print out dfree, Ld, etc.
#define DISPLAYENCODE 0 //print out input, output of encoder
#define DISPLAYDECODE 0 //display decoder output
#define DISPLAYAWGN 0   //show symbols with awgn noise
#define DISPLAYBER 0    //display BER stats
#define DISPLAYUPDATENERR 1   //print nERR after each run

#define DEBUGDECODE 0   //step through decoder

/*GLOBAL VARIABLES, ARRAYS*/
//G(D) = [D^6+D^5+D^3+D^2+1, D^6+D^3+D^2+D+1]
//first index specifies polynomial, second specifies power of D
int G[npolys][orderplus1] = { //{1,1,1},{1,0,1}
                {1,0,1,1,0,1,1},
                {1,1,1,1,0,0,1}
                    };

//states array
//first index is the state, second are the bits describing that
//state, e.g. [1,0,0,0,0,0] corresponds to a single bit in the D^6
//register
int states[nstates][order];
```

```
//transition output array
//first index is destination state,
//second index is originating state,
//third index is output number
int outputs[nstates][nstates][npolys];

//marks if state transition is allowed
//first index is destination state,
//second index is originating state,
//value is 1 if valid transition, 0 else
int validtransition[nstates][nstates];

int dfree, Ld, N, Nb;

char tempchar;


/*PROTOTYPES*/
int smallestnonneg(int *list, int nelements);
int convstats(int display);
int encode(int* input, int inputlength, float** output, int* outputlength,
int display);
int decode(float* received, int receivedlength, int** decodedmsg, int
*decodedlength, int display);
int addAWGN(float signal[], float SNRinv, int len, int display);
float randn(float var);
int calcBER(int input[], int decodedmsg[], int len, int display);

//main
int main(){
      int input[Nin];
    float *output;
    int *decodedmsg;
    int outputlength, decodedlength;
    int m,n, nErr, count;
      float currentSNRinv, currentBER;
      float testSNRs_dB[NTESTSNRS] = TESTSNRS;

      printf("\ng(D) = [");
      for(n=0;n<npolys;n++){
            for(m=0;m<orderplus1;m++){
                  printf("(%d)(D^%d)",G[n][m],m);
                  if(m<order){
                        printf(" + ");
                  }
            }
            printf("  ,  ");
      }
      printf("]\n\n");


      //generate transition matrices, etc.
      convstats(DISPLAYSTATS);

      //display simulation parameters
```

```
      printf("\n___SIMULATION PARAMETERS___\n");
      printf("traceback depth: %d\n",tracebackdepth);
      printf("Nrandn: %d\n",Nrandn);
      if(FASTMSG){
            printf("FASTMSG MODE ON\n");
      }
      printf("NtargetERR: %d\n\n",NtargetERR);

      for(m=0;m<NTESTSNRS;m++){
            nErr = 0;
            count = 0;
            printf("\n__%.2fdB SNR__\n",testSNRs_dB[m]);
            currentSNRinv = pow(10,-0.1*testSNRs_dB[m]);
            srand(time(NULL));
            while(nErr < NtargetERR){
                  //generate new random input vector each time (unless FSTMSG
= 1)
                  if(!count || !FASTMSG){
                        for(n=0;n<Nin;n++){
                              input[n] = rand()%2;
                        }
                  }
                  encode(input, sizeof(input)/sizeof(input[0]), &output,
&outputlength, DISPLAYENCODE);
                  addAWGN(output, currentSNRinv, outputlength, DISPLAYAWGN);
                  decode(output, outputlength, &decodedmsg, &decodedlength,
DISPLAYDECODE);
                  nErr += calcBER(input, decodedmsg, decodedlength,
DISPLAYBER);
                  count++;
                  if(DISPLAYUPDATENERR){
                        printf("nErr = %d  (%d runs of %d)...\n",
                              nErr,count,decodedlength);
                  }
                  //FREE MEMORY
                  free(output);
                  free(decodedmsg);
            }
            currentBER = ((float)nErr)/(count*decodedlength);
            printf("BER: %f         (%d errors, %d runs of %d)\n",currentBER,
nErr, count, decodedlength);
      }

    printf("\n");

    return 0;
}


//decode
int decode(float* received, int receivedlength, int** decodedmsg, int
*decodedlength, int display){

/*finite traceback decoder using Squared Euclidean Distance metric.*/
    float oldweights[nstates];
    float currentweights[nstates];
```

```
    int tracebackmatrix[nstates][tracebackdepth];
    int tracebackptr; //points to most recent col. of tracebackmatrix

    float dist, weight, bestweight;
    int i,j,n,p;

    *decodedlength = receivedlength - 2*tracebackdepth - order; //might try
more complicated padding scheme later
    *decodedmsg = (int*)malloc(*decodedlength*sizeof(int));


    //initialize data structures
    for(i=0;i<nstates;i++){
        oldweights[i] = -1*(i != 0); //-1 means don't consider that state
        currentweights[i] = oldweights[i];
        for(j=0;j<tracebackdepth;j++){
            tracebackmatrix[i][j] = 0;
        }
    }
    tracebackptr = 0;

    for(n=0;n<receivedlength;n++){
            if(DEBUGDECODE){
                  printf("\nweights: [");
                  for(i=0;i<nstates;i++){
                        printf("%f, ",oldweights[i]);
                  }
                  printf("]\nreceived: (");
                  for(i=0;i<npolys;i++){
                        printf("%f, ",received[i*receivedlength + n]);
                  }
                  printf(")");
                  printf("\nTransition weights: ");
                  for(i=0;i<nstates;i++){
                        for(j=0;j<nstates;j++){
                              if(validtransition[i][j]){
                                    printf("(%d<-%d)=(",i,j);
                                    for(p=0;p<npolys;p++){
                                          printf("%d,",outputs[i][j][p]);
                                    }
                                    printf(")   ");
                              }
                        }
                  }
                  scanf("%c",&tempchar);
            }
        //update weight of each state
        for(i=0;i<nstates;i++){ //destination state
            bestweight = -1;
            for(j=0;j<nstates;j++){ //originating state
                if(validtransition[i][j] && (oldweights[j] >= 0)){
                    dist = 0;
                    for(p=0;p<npolys;p++){
                        dist = dist + pow(-1+2*(outputs[i][j][p] != 0)
                                    - received[p*receivedlength + n],2);
                    }
                    weight = oldweights[j] + dist;
```

```
                    if((weight < bestweight) || (bestweight < 0)){
                        bestweight = weight;
                        tracebackmatrix[i][tracebackptr] = j; //record
originating state
                    }
                }
            }
            currentweights[i] = bestweight;
        }
        //at this point we've updated currentweights and tracebackmatrix.
        //output the traceback state and update tracebackptr, and
        //then update oldweights
        bestweight = -1;
        for(i=0;i<nstates;i++){
            weight = currentweights[i];
            if((weight < bestweight) || ((weight >= 0) && (bestweight < 0))){
                bestweight = weight;
                j = tracebackmatrix[i][tracebackptr]; //j=state from which
best state came from
            }
        }

            //traceback
            i=tracebackptr-1;
            if(i<0){
                  i = tracebackdepth-1;
            }
            while(i != tracebackptr){
                  p = tracebackmatrix[j][i]; //trace back one state, store
state in p
                  j = p;      //update j for next iteration
                  i -= 1; //look to next state back
                  if(i<0){
                        i = tracebackdepth-1;
                  }
            }
            //now j holds state transitioned to tracebackdepth transitions
ago, i.e. it holds the answer

        for(i=0;i<nstates;i++){
            oldweights[i] = currentweights[i];
        }

            if((n > 2*tracebackdepth-1) && (n < (2*tracebackdepth +
*decodedlength))){
                  (*decodedmsg)[n-2*tracebackdepth] = j & 1;
              if(display){
              printf("%d ",j & 1);
          }
            }

        tracebackptr++;
        if(tracebackptr >= tracebackdepth){
            tracebackptr = 0; //circular buffer :)
        }
    }
```

```
    return 0;
}
```

//encode

```
int encode(int* input, int inputlength, float** output, int* outputlength,
int display){
/*input is an array of inputs to the convolutional encoder, first
element = first input. output is array of output values. First poly outputs
are at addresses output:output+*outputlength-1, 2nd poly outputs at addresses
output+*outputlength:output+2*(*outputlength)-1, etc.*/
    int sum;
    int n, m, p;

      int augmentedlength;
      int *augmentedinput;

      augmentedlength = inputlength + 2*tracebackdepth; //zeros at beginning
and end
      augmentedinput = (int*)calloc(augmentedlength,sizeof(int));
      for(n=tracebackdepth;n<augmentedlength-tracebackdepth;n++){
            augmentedinput[n] = input[n-tracebackdepth];
      }

    *outputlength = augmentedlength + order; //add leading zeros to "prime"
decoder

    //allocate memory for output vector
    *output = (float*)calloc(*outputlength*npolys,sizeof(float)); //allocate
memory for output array


    //convolution
    for(n=0;n<*outputlength;n++){ //n = output location
        for(p=0;p<npolys;p++){ //consider each output
            sum = 0;
            for(m=0;m<orderplus1;m++){ //summation of shifted terms
                if((n-m >= 0) && (n-m < augmentedlength)){
                    sum += (G[p][m])*(augmentedinput[n-m]);
                }
            }
            (*output)[p*(*outputlength) + n] = 2.0*(sum % 2)-1;
        }
    }

    if(display){
        printf("augmented encoder input: [");
        for(n=0;n<augmentedlength;n++){
            printf("%d, ",augmentedinput[n]);
        }
        printf("]\noutputs (y1,y2,...) = [");
        for(n=0;n<*outputlength;n++){
            printf("(");
            for(p=0;p<npolys;p++){
                printf("%d,",(int)((*output)[p*(*outputlength) + n]));

            }
```

```
            printf("), ");
        }
        printf("]\n\n");
    }

      free(augmentedinput);

    return 0;
}

//convstats
int convstats(int display){

    int i, j, k, n, p; //loop counters
    int sum, valid, bestweight, nsum, nbsum;

    //stuff for finding dfree, etc.
    int colarray[nstates];
    int newcolarray[nstates];
    int sumarray[nstates];
    int nsumarray[nstates];
    int oldnsumarray[nstates];
    int nbsumarray[nstates];
    int oldnbsumarray[nstates];

    //generate array of states
    for(i=0;i<nstates;i++){
        for(j=0;j<order;j++){
            states[i][j] = ((i & (1<<(orderminus1-j))) != 0);
        }
    }

    //generate outputs array
    for(i=0;i<nstates;i++){
        for(j=0;j<nstates;j++){
            valid = 1;
            for(n=0;n<orderminus1;n++){ //check for valid transition
                if(states[i][n] != states[j][n+1]){
                    valid = 0;
                    break;
                }
            }
            if(!valid){
                validtransition[i][j] = 0;
                continue;    //move on to next state transition
            }
            else{
                validtransition[i][j] = 1;
            }
            for(k=0;k<npolys;k++){
                //now compute output value for the valid transition
                sum = (G[k][order])*(states[j][0]);
                for(n=0;n<order;n++){
                    sum += (G[k][n])*(states[i][orderminus1-n]);
                }
                outputs[i][j][k] = sum % 2;
```

```
            }
        }
    }


    //Now we've generated the states and valid transitions and outputs, so
let's find
    //dfree, Ld, N, and Nb

    for(i=0;i<nstates;i++){
        colarray[i] = -1*(i != 0);    //-1 indicates can't transition from
that value
        newcolarray[i] = colarray[i];
        nsumarray[i]= (i == 0);
        nbsumarray[i] = 0;
    }

    Ld = 0;
    while(1){
        //calc cumulative min distance at each state
        for(p=0;p<nstates;p++){
            oldnsumarray[p] = nsumarray[p]; //copy before overwrite
            oldnbsumarray[p] = nbsumarray[p];
        }
        for(i=0;i<nstates;i++){ //looking at state i
            bestweight = (npolys+1)*(Ld+1);    //should be able to beat this
            for(n=0;n<nstates;n++){
                sumarray[n] = -1;
            }
            for(j=0;j<nstates;j++){ //coming from state j
                if(colarray[j] < 0){
                    continue;    //don't transition from this state
                }
                else{
                    if(validtransition[i][j]){
                        sum = colarray[j];
                        for(k=0;k<npolys;k++){
                            sum += outputs[i][j][k];
                        }
                        sumarray[j] = sum;
                        if(sum < bestweight){
                            bestweight = sum;
                        }
                    }
                }
            }

            //for calculation of N
            nsum= 0;
            nbsum = 0;
            for(p=0;p<nstates;p++){
                if(sumarray[p] == bestweight){
                    nsum += oldnsumarray[p];
                    nbsum += oldnbsumarray[p] + states[i][orderminus1];
                }
            }
            nsumarray[i] = nsum;
```

```
            nbsumarray[i] = nbsum;

            if(bestweight < (npolys+1)*(Ld+1)){ //we found a valid transition
to this state
                newcolarray[i] = bestweight;
            }
            else{
                newcolarray[i] = -1; //no allowed to transition to/from this
state
            }
            if(newcolarray[0] == 0){
                newcolarray[0] = -1; //we only want this to be zero for very
first run
            }
        }

        Ld++;    //update Ld
        dfree = newcolarray[0];
        N = nsumarray[0];
        Nb = nbsumarray[0];

        for(i=0;i<nstates;i++){
            colarray[i] = newcolarray[i];
        }

        //check if we have enough information to identify dfree, etc
        newcolarray[0] = -1; //can modify since colarray already updated,
don't need
                             //newcolarray any more this cycle
        if((dfree >= 0) && (smallestnonneg(newcolarray,nstates) > dfree)){
            if(display){
                printf("dfree = %d\n",dfree);
                printf("Ld = %d\n",Ld);
                printf("N = %d\n", N);
                printf("Nb = %d\n", Nb);
                printf("\n");
            }
            break; //we're done
        }
    }

      //print out T(W,I) related matrices if wanted
      //I(i,j) holds i coefficient (input bit errors) of j->i transition
      //W(i,j) holds w coefficient (output weight) of j->i transition
      //format should be easy to process in Octave or MATLAB
      if(GEN_TWI){
            printf("I = ...\n[");
            for(i=0;i<nstates;i++){ //i = dest. state
                  for(j=0;j<nstates;j++){ //j = src. state
                        printf("%d ",(validtransition[i][j] & i & 1));
                  }
                  if(i+1 != nstates){
                        printf(";...\n");
                  }
            }
            printf("];\n\n");
```

```
            printf("W = ...\n[");
            for(i=0;i<nstates;i++){ //i = dest. state
                  for(j=0;j<nstates;j++){ //j = src. state
                        sum=0;
                        for(p=0;p<npolys;p++){
                              sum += outputs[i][j][p];
                        }
                        printf("%d ",sum*validtransition[i][j]);
                  }
                  if(i+1 != nstates){
                        printf(";...\n");
                  }
            }
            printf("];\n\n");

            printf("V = ...\n[");   //valid transition indicator
            for(i=0;i<nstates;i++){ //i = dest. state
                  for(j=0;j<nstates;j++){ //j = src. state
                        printf("%d ",validtransition[i][j]);
                  }
                  if(i+1 != nstates){
                        printf(";...\n");
                  }
            }
            printf("];\n\n");
      }

    return 0;
}

//randn
float randn(float var){
//generate a random variable from a normal distribution with variance var
//r.v. generated using law of large numbers
      int i, sum;
      sum = 0;
      for(i=0;i<Nrandn;i++){
            sum += rand()%2;
      }
      return 2.0*sqrt(var)*(((float)sum)/sqrt(Nrandn)-sqrt(Nrandn)*.5);
}

//smallestnonneg
int smallestnonneg(int *list, int nelements){
//return smallest non-negative value in list[]
    int i, temp;
    temp = -1; //initialize to -1, if this value is returned, there was a
problem
    for(i=0;i<nelements;i++){
        if((list[i] >= 0) && (temp < 0)){
            temp = *(list+i);
        }
        if((list[i] < temp) && (list[i] >= 0)){
            temp = list[i];
        }
    }
```

```
    return temp;
}

//calcBER
int calcBER(int input[], int decodedmsg[], int len, int display){
      int i;
      int sum=0;
      for(i=0;i<len;i++){
            sum += (input[i] != decodedmsg[i]);
      }
      if(display){
            printf("\n\nBER: %f   (%d errors, %d bits)\n",((float)
(sum))/len,sum,len);
      }
      return sum;
}

//addAWGN
int addAWGN(float signal[], float SNRinv, int len, int display){
//assumes signal has unit power
      int i,p;
      for(i=0;i<len*npolys;i++){
            signal[i] += randn(SNRinv);
      }
      if(display){
            for(i=0;i<len;i++){
                  printf("(");
                  for(p=0;p<npolys;p++){
                        printf("%f, ",signal[i+p*len]);
                  }
                  printf(")\n");
            }
      }
      return 0;
}
```

2) MATLAB Code (transfer function bound computation, plotting routines)

combined_plot.m

```
snr14 = [2,3,4];
ber14 = [0.023110,0.003990,0.000407];

snr28 = [2,3,4];
ber28 = [0.007270,0.000935,0.000047];

snr35 = [2,3,4];
ber35 = [0.007470,0.000433,0.000024];

snr56 = [2,3,4];
```

```
ber56 = [0.004910,0.000357,0.000026];

snr100 = [4.0,4.1,4.2,4.35,4.5,4.8];
ber100 = [0.000018,0.000010,0.000014,0.000006,2.9412e-06,1.3831e-6];

snrtheory = 1:0.05:6;
bertheory = 36*qfunc(sqrt(10*power(10,0.1*snrtheory)));

snrtbnd = 1:0.05:6;
for m = 1:length(snrtbnd)
      bertbnd(m) = calc_transfer_bound(.5*10^(.1*snrtbnd(m)));
end

figure(1);
hold off
semilogy(snrtheory,bertheory,'b-');
hold on
semilogy(snrtbnd,bertbnd,'b-');
semilogy(snr14,ber14,'ko-','markersize',15);
semilogy(snr28,ber28,'kx-','markersize',15);
semilogy(snr35,ber35,'go-','markersize',15);
semilogy(snr56,ber56,'mo-','markersize',15);
semilogy(snr100,ber100,'r*','markersize',20);
hold off
grid on
legend('approx.','Transfer Function Bound','depth 14','depth 28','depth
35',...
      'depth 56','depth 100')
xlabel('SNR (dB)')
ylabel('BER')
title('BER vs. SNR, g(D) = [1+D^2+D^3+D^5+D^6, 1+D+D^2+D^3+D^6]')
```

calc_transfer_bound.m

```
function result = calc_transfer_bound(EsNo)

%EsNo = 1; %Es/No
NSTATES = 64;
k=1;
dfree = 10;

%generate I,W,V matrices
gen_IWV;
%I = [0 0;1 1];
%W = [0 2;2 2];
%V = [1 1;1 1];

%store as [w,i]
IVal = 1;
WVal = exp(-EsNo);

dw = W(1,1); di = I(1,1);
cw = W(1,2:end); ci = I(1,2:end);
```

```
bw = W(2:end,1); bi = I(2:end,1);
Aw = W(2:end,2:end); Ai = I(2:end,2:end);


AVal = Aw;
bVal = bw;
cVal = cw;

dVal = (IVal^di)*(WVal^dw)*V(1,1);

for m = 1:NSTATES-1
      for n = 1:NSTATES-1
            AVal(m,n) = (IVal^Ai(m,n))*(WVal^Aw(m,n))*V(m+1,n+1);
      end
end

for m = 1:NSTATES-1
      bVal(m) = (IVal^bi(m))*(WVal^bw(m))*V(m+1,1);
      cVal(m) = (IVal^ci(m))*(WVal^cw(m))*V(1,m+1);
end

AprimeVal = Aw;
bprimeVal = bw;
cprimeVal = cw;

dprimeVal = di*(IVal^(di-1))*(WVal^dw)*V(1,1);

for m = 1:NSTATES-1
      for n = 1:NSTATES-1
            AprimeVal(m,n) = Ai(m,n)*(IVal^(Ai(m,n)-
1))*(WVal^Aw(m,n))*V(m+1,n+1);
      end
end

for m = 1:NSTATES-1
      bprimeVal(m) = bi(m)*(IVal^(bi(m)-1))*(WVal^bw(m))*V(m+1,1);
      cprimeVal(m) = ci(m)*(IVal^(ci(m)-1))*(WVal^cw(m))*V(1,m+1);
end

result = 1/k*qfunc(sqrt(2*dfree*EsNo))*(dprimeVal +
cVal*inv(eye(length(cVal))-AVal)*bprimeVal +...
      cprimeVal*inv(eye(length(cVal))-AVal)*bVal +...
      cVal*inv(eye(length(cVal))-AVal)*AprimeVal*inv(eye(length(cVal))-
AVal)*bVal)*...
      exp(dfree*EsNo);


end
```

gen_IWV.m

```
I = ...
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
```

```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
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0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
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```
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0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 ];

W = ...
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
0 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 2 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ;...
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
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0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
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